Genetic Algorithm Optimization for Tensioning over Double Annular Domain

Masayuki Ishihara\(^1\帮助企业*, Hiroki Murakami\(^2\) and Yoshihiro Ootao\(^3\)

\(^1,\, ^3\) Osaka Prefecture University, 1-1, Gakuen, Naka, Sakai, Osaka 599-8531, Japan
\(^2\) Isuzu Motors Limited, 6-26-1, Minami-Oi, Shinagawa, Tokyo 140-8722, Japan

* Corresponding author, e-mail: (ishihara@me.osakafu-u.ac.jp)

(Received: 8-11-13; Accepted: 9-12-13)

**Abstract:** With the aim of achieving stable operation of circular saws, this study finds the solution for the optimization problem that involves choosing a set of tensioning parameters in a rotating circular saw that is subjected to both a local temperature distribution and the in-plane plastic strain over a double annular domain. The solution for the in-plane forces is obtained on the basis of plate bending theory, and modal analysis for the out-of-plane behavior affected by the in-plane forces is performed. Numerical calculations are performed to investigate the effects of tensioning over the double annular domain on the natural frequencies. The optimization problem to maximize the natural frequency of the most critical mode with respect to the intensities, locations, and widths of tensioning is solved using a genetic algorithm, and the optimal tensioning parameters are determined at computational costs that are considerably lower than those required for 100% inspection.

**Keywords:** Circular saw, Tensioning, Natural frequency, Optimization problem, Genetic algorithm.

1. Introduction:

Circular saws are indispensable tools for machining. They are normally subjected to a thermal load resulting from the friction between the blade and the workpiece. This thermal load generally produces compressive in-plane forces, which in turn change various dynamic characteristics such as natural frequencies \([1]\). Due to the change in the dynamic characteristics, vibrations that are excited by the interaction between the teeth of the saw blade and the workpiece tend to become unstable and degrade the working accuracy of the saw \([8]\). In order to circumvent this problem, circular saws are
often subjected to a prestressing procedure called roll tensioning, wherein the saws are sandwiched between two rollers under a compressive force and then rotated slowly to induce plastic deformation and introduce in-plane tension. Plastic deformation occurs around the rolled region. Reviews on tensioning studies can be found in the literature [14, 11]. Numerous studies on tensioned saws have been conducted [10, 12, 13, 6, 7, 9].

As mentioned in the above studies, the natural frequency of a circular saw has a significant impact on its stability; in general, the stability can be improved by increasing the natural frequency. Tensioning serves this purpose. Therefore, in a previous study [5], we performed theoretical analysis of the thermoelastoplastic field and the resulting natural frequency of a rotating, thermally loaded circular saw that was tensioned over a single annular domain, and we investigated the effects of tensioning on the in-plane forces and natural frequencies. Moreover, we extended the study to the case where a saw undergoes tensioning over a double annular domain, assuming actual conditions [4].

The efficiency of tensioning depends on the control of the distribution of the applied plastic deformation, which requires skill and years of experience on the part of the operator. Moreover, once a saw is subjected to tensioning, it is nearly impossible to modify the tensioning parameters such as intensities, locations, and widths. Therefore, it is important to estimate the effect of tensioning by performing simulations before actual tensioning is performed.

In order to determine the optimal combination of tensioning parameters for the assumed environments that a particular saw is likely to encounter, it is necessary to perform numerical calculations for various combinations of many types of tensioning parameters. For example, various combinations of three parameters, i.e., intensity, location, and width, must be considered for a single tensioned saw, and those of six parameters, i.e., two intensities, two locations, and two widths, must be considered for a double tensioned saw. In other words, the number of dimensions in the optimization problem can be quite high. Moreover, in the previous study [4], the effects of the in-plane forces on the out-of-plane motion were taken into account, unlike in the case of ordinary plate bending theory. From the results of this study [4], it was found that the effects of tensioning parameters on dynamic characteristics such as natural frequencies exhibited high nonlinearity and that the computational cost to obtain the optimal dynamic characteristics increased. The high number of dimensions and computational cost made it impossible to determine the optimal combination of tensioning parameters using ordinary nonlinear programming methods.

Genetic algorithms have been widely used for other optimization problems in which the objective function exhibits high nonlinearity with respect to many types of design variables [3]. The genetic algorithm is one of the heuristic search techniques that imitate the manner in which creatures adapt to their environments over generations, through genetic operations. The algorithm is effective in solving optimization problems with a high degree of nonlinearity, as mentioned above.

In this study, therefore, we investigated the use of the genetic algorithm to solve the optimization problem for the tensioning parameters in a rotating circular saw under a thermal load that is tensioned over a double annular domain. The analytical model that we used is a rotating annular disk that is subjected to both a local temperature distribution arising from the thermal load, which is caused by blade-workpiece friction, and the in-plane plastic strain introduced by tensioning over a double annular domain. On the basis of the analytical results for in- and out-of-plane behaviors, we performed numerical calculations to investigate the effects of various combinations of the tensioning parameters on the natural frequencies. We then used the genetic algorithm to solve the optimization problem to maximize the natural frequency of the most critical mode with respect to the intensities, locations, and widths of tensioning.
2. Theoretical Analysis:

2.1. Problem

The theoretical analysis is shown in Fig. 1. This model includes an annular disk with inner and outer radii \( r_i \) and \( r_o \), respectively, and thickness \( h \); the annular disk rotates around the \( z \)-axis at angular velocity \( \omega_k \). The mass density, Young’s modulus, Poisson’s ratio, and coefficient of linear thermal expansion are denoted by \( \rho \), \( E \), \( \nu \), and \( \alpha \), respectively. The cylindrical coordinate system \((r, \theta, z)\) is defined in the frame of reference of the rotating disk. Time is denoted by \( t \). The disk is clamped at the inner rim and is free of traction at the outer rim. The disk has an arbitrary temperature distribution of \( T = T_0 f(r) \), resulting from the friction between the blade and the workpiece, where \( T_0 \) denotes a representative temperature. Moreover, in-plane plastic strains \( \varepsilon_1 f_1(r) \) and \( \varepsilon_2 f_2(r) \) are assumed to be induced by tensioning over a double annular domain, where \( \varepsilon_k \), \( r_k \), and \( b_k \) \((k = 1,2)\) denote the intensities, locations, and widths of tensioning, respectively.

\[ T = T_0 f(r), \]

\[ \varepsilon_1 f_1(r), \]

\[ \varepsilon_2 f_2(r), \]

\[ \theta \]

\[ \omega_k \]

\[ r \]

\[ f_1(r) \]

\[ f_2(r) \]

\[ f_3(r) \]

\[ b_1 \]

\[ b_2 \]

\[ T_0 \]

\[ f_1^* \]

\[ f_2^* \]

\[ f_3^* \]

\[ \varepsilon_1 \]

\[ \varepsilon_2 \]

\[ \varepsilon_3 \]

Fig. 1: Analytical model

2.2. Analysis of In-Plane Resultant Forces

To simplify the analysis, non-dimensional variables are introduced as

\[ (r, r_1, r_2, b_1, b_2) = \left( \frac{r}{r_o}, r_1, r_2, b_1, b_2 \right), \]

\[ l = \sqrt{\frac{E h^3}{12(1-\nu^2)\rho r_o^4 t}}, \]

\[ \left( N_{rr}, N_{\theta \theta} \right) = \left( \frac{12(1-\nu^2)r_o^2}{E h^3} N_{rr}, \frac{12(1-\nu^2)r_o^2}{(E h^2)\alpha_k} N_{\theta \theta} \right), \]

\[ w = \sqrt{12(1-\nu^2)}h \omega_k, \]

\[ \left( T_0^*, \varepsilon_1^*, \varepsilon_2^* \right) = \left( 12(1-\nu^2)(r_o/h)^2(\alpha T_0^*, \varepsilon_1^*, \varepsilon_2^*) \right) \]

Where \( N_{rr} \) and \( N_{\theta \theta} \) denote the in-plane resultant forces in the radial and hoop directions, respectively, and \( w \) denotes the deflection.

In a previous study [4], the analytical solution of the in-plane resultant forces under the boundary conditions stated in Subsection 2.1 was formulated as

\[ N_{rr} = 2C_1 f_1 + C_2 f_2 \left[ \frac{\rho}{8} - \left( 3 + \nu \right) \left( \frac{\rho}{8} \right) \right], \]

\[ N_{\theta \theta} = 2C_1 - C_2 \left[ \frac{\rho}{8} - \left( 3 + \nu \right) \left( \frac{\rho}{8} \right) \right] + \frac{\rho}{8} \left[ f_1^*(r) + f_2^*(r) \right], \]

\[ + \frac{\rho}{8} \left( -f_1^*(r) + f_1(r) + f_2(r) + f_2^*(r) \right], \]

\[ (1) \]

\[ (2) \]
where \( f_k^*(r) \equiv \left[ \int f_k^* \left( \frac{d\theta}{r} \right) d\theta \right]^2 \) \((k = 1, 2)\). The definitions of \( C_{F1} \) and \( C_{F2} \) can be obtained from the previous study [4]. For the distribution of plastic strains shown in Fig. 1, \( f_k(r), f_k^*(r), C_{F1}, \) and \( C_{F2} \) are now calculated as

\[
\begin{align*}
&f_k(r) = H(r - \left( r_k - b_k/2 \right)) - H(r - \left( r_k + b_k/2 \right)), \\
&f_k^*(r) = (1/2) \left[ 1 - \left( r_k - b_k/2 \right)^2 \right]^{1/2} \left[ H(r - \left( r_k - b_k/2 \right)) - H(r - \left( r_k + b_k/2 \right)) \right] + \left( r_k b_k / r^2 \right) \cdot \left\{ H(r - \left( r_k + b_k/2 \right)) - H(r - 1) \right\} : k = 1, 2
\end{align*}
\]

with the Heaviside unit function \( H(*) \), and

\[
\begin{align*}
2C_{F1} &= R_{01} \frac{d^2}{dr^2} + C_{T1} T_0 + \sum_{k=1}^{2} C_{\epsilon_{1k}} \frac{d^2}{d\theta^2}, & C_{F2} &= \frac{1}{r^2} \left( C_{R2} \frac{d^2}{dr^2} + C_{T2} T_0 + \sum_{k=1}^{2} C_{\epsilon_{2k}} \frac{d^2}{d\theta^2} \right), \\
C_{\epsilon_{1k}} &= \frac{\rho \nu b_k (1 + \nu) / [1 + \nu + (1 - \nu) r_k^2]}{1 + \nu + (1 - \nu) r_k^2} : k = 1, 2
\end{align*}
\]

where the definitions of \( C_{R1}, C_{R2}, C_{T1}, \) and \( C_{T2} \) are the same as those presented in the previous study [4].

### 2.3. Modal Analysis of Flexural Vibration

Modal analysis for the flexural vibration that is affected by the in-plane forces is presented. The procedures of modal analysis for a different formula for \( \hat{N}_{rr} \) and \( \hat{N}_{\theta\theta} \) in the previous study [4] are also applicable to the present study and are summarized as follows.

The deflection is expressed as

\[
\hat{W} = \phi_n(r) \exp(in\theta) \exp(i\phi_n r),
\]

where \( \phi_n(r) \) denotes the mode function in the radial direction and \( n \), which is referred to as the number of nodal diameters, denotes the number of sinusoidal waves in the circumferential direction. By substituting Eq. (5) into the fundamental equation for deflection and using the boundary conditions stated in Subection 2.1, we obtain the governing equations for mode function \( \phi_n(r) \). The Galerkin method [2] is used to solve the governing equations thus obtained. The mode function is expressed as a series of up to \( M \) terms:

\[
\phi_n(r) = \sum_{m=1}^{M} \phi_{nm}(r),
\]

where the mode function for a plate in the absence of in-plane resultant forces \( \hat{N}_{rr} \) and \( \hat{N}_{\theta\theta} \) is chosen as trial function \( \phi_{nm}(r) \). Thus, from our previous study [4], we have

\[
\phi_{nm}(r) = C_{1nm} \int_{0}^{r} J_n \left( \sqrt{\int_{0}^{\theta_{nm}} r} \right) + C_{2nm} \int_{0}^{r} Y_n \left( \sqrt{\int_{0}^{\theta_{nm}} r} \right) + C_{3nm} K_n \left( \sqrt{\int_{0}^{\theta_{nm}} r} \right) + C_{4nm} Y_n \left( \sqrt{\int_{0}^{\theta_{nm}} r} \right),
\]

where \( J_n(*) \) and \( Y_n(*) \) denote the Bessel functions of the first and second kind, respectively, of order \( n \), and \( I_n(*) \) and \( K_n(*) \) denote the modified Bessel functions of the first and second kind,
respectively, of order \( n \). In addition, \( \omega_{nm} \) denotes the natural frequency of the plate in the absence of in-plane forces and is obtained as the \( m \)-th smallest positive solution of
\[
\det(A_{nm}) = 0.
\] (8)

The definitions of \( A_{nm} \) and \( C_{iam} (i = 1, 2, 3, 4) \) are as shown in the previous study [4]. It should be noted that \( m \) denotes the number of nodal circles. After applying the Galerkin method, we obtain the equation for mode coefficient \( \phi_{nm} \) as follows:
\[
[M_n](\Phi_n) = \omega_{nm}^2 (\Phi_n),
\] (9)
where
\[
(M_n)_{ij} = \delta_{ij} \omega_{nm}^2 - \int_1^1 \left[ N_{r} \frac{d^2 f_{nj}}{dr^2} + N_{\theta} \left( \frac{1}{r} \frac{df_{nj}}{dr} - \frac{n^2}{r^2} f_{nj} \right) \right] f_{mj} dr,
\] (10)

Thus, by solving the eigenvalue problem given by Eq. (9), we obtain natural frequencies \( \omega_{nm} \) and eigenvectors \( \{\Phi_n\} \), and hence, eigenmodes \( \phi_n(t) \) through Eq. (6). Here, \( \omega_{nm} \) denotes the \( m \)-th smallest positive solution of \( \omega_{nm} \). From Eqs (9) and (10), we find that the natural frequencies are dependent on in-plane resultant forces \( N_r \) and \( N_{\theta} \). In-plane resultant forces \( N_r \) and \( N_{\theta} \) are in turn dependent on \( k_{\varepsilon} \), \( k_r \), and \( k_b \) \((k = 1, 2)\), as stated in Subsection 2.2. Therefore, the natural frequencies are dependent on intensities \( k_{\varepsilon} \), locations \( k_r \), and widths \( k_b \) \((k = 1, 2)\) of tensioning.

3. Numerical Calculation:

As stated in Section 1, the stability of a circular saw can generally be improved by increasing the natural frequency. Therefore, this section presents the numerical calculations used to investigate the effects of varying the tensioning parameters on the resulting natural frequencies. Moreover, in order to achieve the stable operation of the saw, the optimization problem to maximize the natural frequency of the most critical mode is solved using the genetic algorithm.

The conditions for the calculations are as follows. Poisson’s ratio and the non-dimensional inner radius are assumed to be \( \nu = 0.25 \) and \( \bar{r} = 0.3 \), respectively. The saw is considered to be rotating at a non-dimensional angular velocity of \( \omega_R = 3 \). The temperature is considered to be distributed over the outer edge as \( f_j(t) = H(t - 0.95) - H(t - 1) \), with non-dimensional temperature \( \bar{T}_0 = 100 \). With regard to the natural frequencies, we focus on the modes with the nodal diameter \( m = 1 \) and nodal circles \( n = 0 \sim 5 \) as the modes that would be critical in practice.

3.1. Effect of Tensioning over Double Annular Domain

Figure 2 shows the variations in the natural frequencies with the tensioning locations. As shown in the figure, the variations are dependent on the modes. In addition, the most critical mode, which minimizes the natural frequency with respect to the modes for a given combination of tensioning locations, changes depending on the combination: for example, in Fig. 2 (a), the most critical mode exhibits the transition from \( n = 0 \) through \( n = 1 \) and \( n = 2 \) to \( n = 3 \) as \( h_2 \) increases.
In Fig. 2, variations in the natural frequency of the most critical mode, \( \min_{k \in \mathbb{N}} (\omega_{kn}) \), with tensioning location \( l_2 \) are described by the connected curve composed of the curves with the lowest value of the ordinate. In view of this, Fig. 2 (a) shows that the natural frequency of the most critical mode reaches the maximum value \( 3.7 \) at \( \omega_{k1} \approx 77.0 \) while Fig. 2 (b) shows that the natural frequency reaches the maximum value \( 3.7 \) at \( \omega_{k1} \approx 59.0 \). Figures 2 (a) and (b) suggest that, in order to achieve stable operation of the saw, the combination of tensioning parameters should be chosen properly. Moreover, it is found that the different combinations \((l_1, l_2)\) give the same maximum value \( 3.7 \), which suggests the existence of multiple solutions to the optimization problem treated in the following subsections.

![Graph showing variations in natural frequencies with combination of tensioning locations](image)

(a) \( l_1 = 0.6 \)  
(b) \( l_1 = 0.7 \)

**Fig. 2:** Variations in natural frequencies with combination of tensioning locations  
\( \{l_1 = 500, b_1 = 0.05; l_2 = 250, b_2 = 0.05\} \)

### 3.2. Optimization Problem Using Genetic Algorithm

In order to achieve stable operation of the saw, we need to solve the optimization problem that maximizes the most critical natural frequency with respect to the tensioning parameters. This frequency is also the minimum natural frequency with respect to the modes. The problem is described as follows:

\[
\begin{align*}
\text{Maximize} & \quad f(l_1, l_2, \epsilon_{12}, \epsilon_{13}, \epsilon_{23}) = \min_{k \in \mathbb{N}} (\omega_{kn}) \\
\text{subject to} & \quad L_{rk} \leq \epsilon_{k} \leq U_{rk}, \quad L_{rk} \leq l_1 \leq U_{rk}, \quad L_{bk} \leq b_1 \leq U_{bk} (k = 1, 2),
\end{align*}
\]

where \( L_{idk} \) and \( U_{adk} \) (\( id = \epsilon, r, b; k = 1, 2 \)) denote the lower and upper boundaries of design variables \( \epsilon_{k} \), \( l_1 \), and \( b_1 \), respectively.

Because the most critical mode depends on the tensioning parameters, as illustrated in Fig. 2, various modes should be reviewed in observing the natural frequencies. Moreover, because the natural frequencies are obtained numerically using Eqs (2)-(4) and Eqs (7)-(10) owing to the absence of explicit formulas, a considerable computational cost is required to calculate the frequencies for a given set of tensioning parameters. Figures 2 (a) and (b), which show the dependence of the
frequency on only tensioning location \( \hat{b}_2 \), are obtained by 100\% inspection, i.e., by repeating the abovementioned method for as many tensioning locations \( \hat{b}_2 \) as possible. However, it is almost impossible to investigate the frequency dependence on all six tensioning parameters (intensities \( \hat{k}_1 \) and \( \hat{k}_2 \), locations \( \hat{h}_1 \) and \( \hat{h}_2 \), and widths \( \hat{b}_1 \) and \( \hat{b}_2 \)) using this method. However, when an ordinary nonlinear programming method is employed for the optimization problem to avoid the computation costs of 100\% inspection, there is a potential for the method’s results to fall into a local solution: for example, in Fig. 2 (a), the local maximum of \( \min_{0 \leq \omega \leq 150} (\hat{b}_n) \) can be found not only at \( \hat{b}_2 \cong 0.77 \) but also at \( \hat{b}_2 = 0.325 \). In order to overcome these difficulties, we employed the genetic algorithm for the optimization problem.

In this technique, the values of \( \hat{k}_1, \hat{k}_2, \) and \( \hat{b}_k \) \((k=1, 2)\) are treated as a string of binary bits, which is referred to as a chromosome. By denoting the digit lengths for the binary bits for \( \hat{k}_1, \hat{k}_2, \) and \( \hat{b}_k \) as \( n_{rk}, n_{rk}, \) and \( n_{rk} \), respectively, chromosome \( c \) is expressed as

\[
c = \left\{ b_{11}^r, b_{12}^r, A, b_{21}^r, b_{22}^r, A, b_{31}^r, b_{32}^r, A, b_{41}^r, b_{42}^r, A, b_{51}^r, b_{52}^r, A, b_{61}^r, b_{62}^r, A \right\} \quad (12)
\]

where \( \{ b_{11}^r, b_{12}^r, A, b_{21}^r, b_{22}^r, A, b_{31}^r, b_{32}^r, A, b_{41}^r, b_{42}^r, A, b_{51}^r, b_{52}^r, A, b_{61}^r, b_{62}^r, A \} \) are the binary expressions of \( \hat{k}_1, \hat{k}_2, \) and \( \hat{b}_k \), respectively, and are related to the corresponding decimal values as

\[
\begin{align*}
\hat{k}_1 &= L_{k1} + \frac{U_{k1} - L_{k1}}{2^{n_{rk1}} - 1} \sum_{l=1}^{n_{rk1}} (b_{11}^l \cdot 2^{n_{rk1} - l}), \\
\hat{k}_2 &= L_{k2} + \frac{U_{k2} - L_{k2}}{2^{n_{rk2}} - 1} \sum_{l=1}^{n_{rk2}} (b_{22}^l \cdot 2^{n_{rk2} - l}), \\
\hat{b}_k &= L_{b2} + \frac{U_{b2} - L_{b2}}{2^{n_{rk2}} - 1} \sum_{l=1}^{n_{rk2}} (b_{22}^l \cdot 2^{n_{rk2} - l}) \quad (k = 1, 2)
\end{align*}
\]

(13)

It should be noted that design variables \( \hat{k}_1, \hat{k}_2, \) and \( \hat{b}_k \) can take \( 2^{n_{rk1}}, 2^{n_{rk2}}, \) and \( 2^{n_{rk2}} \) variations, respectively.

The procedure of the genetic algorithm is as follows. Let us consider a group composed of \( N_{\text{population}} \) chromosomes \( C_i = \{ c_{1j}^i, c_{2j}^i, A, c_{nj}^{i_{\text{permut}}} \} \), each element of which is expressed in the format shown by Eq. (12), in the \( i \)-th generation. Here, \( c_{ij}^i \) \((j = 1, 2, A, N_{\text{population}})\) is referred to as an individual and \( N_{\text{population}} \) as the population of the individuals. The variable \( c_{ij}^i \) is then transformed into decimal design variables \( \hat{k}_1, \hat{k}_2, \) and \( \hat{b}_k \) \((k = 1, 2)\) using Eqs (12) and (13). In addition, the objective function defined by Eq. (11), \( f(\hat{k}_1, \hat{k}_2, \hat{b}_1, \hat{b}_2, \hat{b}_3, \hat{b}_4) \), is evaluated by the method stated in Section 2 as

\[
F_{ij} \equiv f\left[ \hat{k}_1(c_{ij}), \hat{k}_2(c_{ij}), \hat{b}_1(c_{ij}), \hat{b}_2(c_{ij}), \hat{b}_3(c_{ij}), \hat{b}_4(c_{ij}) \right],
\]

(14)

which is referred to as fitness in genetic algorithm terms. The group composed of \( N_{\text{population}} \) chromosomes in the \((i + 1)\)-th generation, \( C_{i+1} = \{ c_{1j}^{i+1}, c_{2j}^{i+1}, A, c_{nj}^{i+1_{\text{permut}}} \} \), is generated by a series of genetic operations, that is, selection, crossover, and mutation, in this order. First, the initial group of
chromosomes, $C_0 = \{c_1^0, c_2^0, \Lambda, c_{N_{population}}^0\}$, is randomly generated using a uniform random number generator. Next, in the selection operation, elite preservation and roulette selection are performed: the individuals with the $N_{elite}$ highest fitness values in the $i$-th generation are preserved as elite; the remaining $(N_{population} - N_{elite})$ individuals are selected from all the individuals in the $i$-th generation in accordance with a probability that is proportional to the fitness of each individual. The group composed of $N_{population}$ chromosomes that is obtained in this manner then undergoes the crossover operation: $N_{elite}$ individuals preserved as elite are kept unchanged; from the remaining $(N_{population} - N_{elite})$ selected individuals, some pairs of individuals are selected with probability $p_{crossover}$, and, among each selected pair, a string of binary bits is exchanged in portions that are randomly selected by using a uniform random number generator. The group that underwent the crossover operation is then subjected to mutation: all bits of all the chromosomes other than elite chromosomes are independently subjected to an exchange of 0 and 1 with a slight probability of $p_{mutation}$. This method increases overall fitness and generates $C_{i+1} = \{c_1^{i+1}, c_2^{i+1}, \Lambda, c_{N_{population}}^{i+1}\}$. These procedures are then iterated for each generation.

### 3.3. Verification of Genetic Algorithm

In order to verify whether the genetic algorithm functions properly, we first solved the optimization problem described by Eq. (11) only with respect to tensioning intensity $I_2$, by setting $L_{e1} = U_{e1} = 500$, $L_{i1} = U_{i1} = 0.6$, $L_{b1} = U_{b1} = 0.05$, $L_{e2} = U_{e2} = 250$, and $L_{b2} = U_{b2} = 0.05$. The selected parameters are $L_{r2} = 0.325$, $U_{r2} = 0.975$, $n_{r2} = 10$; $N_{population} = 3$, $N_{elite} = 1$; $p_{crossover} = 0.7$, $p_{mutation} = 0.05$. (15)

Figure 3 shows the variations in the individuals’ distributions with the generations, $I_2(c_j^i)$ ($j = 1, 2, 3$) (denoted by 3 dots, some of which may overlap, for each generation $i$). This figure also shows the maximum and average fitness values, defined respectively by

$$F_{max}^i = \max_{j=1,2,3,N_{population}} (F_j^i), \quad F_{avg}^i = \frac{1}{N_{population}} \sum_{j=1}^{N_{population}} F_j^i.$$ (16)

Figure 3 shows that the individuals’ $I_2$, initially distributed at random in the range $L_{r2} \leq I_2 \leq U_{r2}$, gather at $I_2 = 0.7660$, whereas $F_{max}^i$ increases and converges on $F_{max}^i \equiv 7.311$ after many generations. This behavior agrees with the result shown in Fig. 2 (a), in which $f(500,0.6,0.05;250,I_2,0.05)(= \min_{0 \leq n \leq 5} (\partial_n \omega_1)) \equiv 7.3$ at $I_2 \equiv 0.77$. This agreement shows that the genetic algorithm succeeds in determining the solution for the optimization problem described by Eq. (11). Moreover, the genetic algorithm’s search for a solution is performed at a much lower computational cost than that of 100% inspection. Although the solution is obtained from $2^{10}(=1024)$ candidates of $I_2$, as shown in Eqs (13) and (15), the calculation (the number of individuals (3) multiplied by the number of generations (30)) to obtain Fig. 3 is performed only 90 times. Such a saving in the computational cost is significantly advantageous for the optimization problem with respect to multiple design variables, as discussed in subsequent subsections. After the convergence on $I_2 \equiv 0.7660$ is substantially accomplished, as shown in Fig. 3, outliers sometimes appear as a result of the mutation operation. Because of such outliers, the genetic algorithm is also
expected to be applicable to an optimization problem in which the fitness function has multiple local maxima.

Fig. 3: Variations in individuals’ $l_i^r$ and fitness with generation $(\delta_1^r = 500, \delta_1^l = 0.6, \delta_1^b = 0.05; \delta_2^r = 250, \delta_2^b = 0.05)$

3.4. Optimization with Combination of Tensioning Locations

Next, the optimization problem described by Eq. (11) is solved with respect to the combination of tensioning locations $l_i^r$ and $l_i^b$, by setting $L_{r1} = U_{r1} = 500$, $L_{b1} = U_{b1} = 0.05$, $L_{r2} = U_{r2} = 250$, and $L_{b2} = U_{b2} = 0.05$. The selected parameters are

$$L_{r1} = 0.325, U_{r1} = 0.975, n_{r1} = 10; L_{r2} = 0.325, U_{r2} = 0.975, n_{r2} = 10;$$

$$N_{\text{population}} = 20, N_{\text{elite}} = 2; p_{\text{crossover}} = 0.7, p_{\text{mutation}} = 0.05$$

(a) $i = 0$

(b) $i = 10$
Figure 4 shows the distributions of individuals’ \((l_1, l_2)\) and corresponding fitness \(F_j\) in various generations. Figure 5 shows the variations in the maximum and average fitness values defined by Eq. (16) with generations. Figure 4 shows that the individuals’ \((l_1, l_2)\) that are initially distributed at random in the range of \(\{(l_1, l_2) \mid L_1 \leq l_1 \leq U_1, L_2 \leq l_2 \leq U_2\}\) gather in the domain of \(\{(l_1, l_2) \mid 0.6 \leq l_1 \leq 0.7, 0.7 \leq l_2 \leq 0.8\}\). In addition, \(F_j^{\max}\), the largest length of the downward lines in each figure, increases with the number of generations. By inspecting Figs 4 and 5, the solution to the problem is found to be \((l_1, l_2) \equiv (0.6192, 0.7463)\) which gives \(f(500, l_1, 0.05; 250, l_2, 0.05) \equiv 7.344\). The solution is obtained from \(2^{10} \times 2^{10} (= 1048576)\) candidates of \((l_1, l_2)\), as shown in Eqs (13) and (17); however, the calculation (the number of individuals (20) multiplied by the number of generations (30)) to obtain Fig. 5 is performed only 600 times. Thus, the computational cost is reduced to \(1/1748(\equiv 600/(2^{10} \times 2^{10}))\) of the cost of 100% inspection, which shows an even greater advantage for a solution search using the genetic algorithm.
3.5. Optimization with Combination of Tensioning Intensities

Next, the optimization problem described by Eq. (11) is solved with respect to the combination of tensioning intensities $\varepsilon_1$ and $\varepsilon_2$, by setting $L_{r1} = U_{r1} = 0.6$, $L_{p1} = U_{p1} = 0.05$, $L_{r2} = U_{r2} = 0.7$, and $L_{p2} = U_{p2} = 0.05$. The selected parameters are

$$L_{\varepsilon1} = 0, U_{\varepsilon1} = 1000, n_{\varepsilon1} = 10; L_{\varepsilon2} = 0, U_{\varepsilon2} = 1000, n_{\varepsilon2} = 10;$$

$$N_{\text{population}} = 5, N_{\text{elite}} = 1; p_{\text{crossover}} = 0.7, p_{\text{mutation}} = 0.05.$$  \hfill (18)

Figure 6 shows the distributions of individuals' $(\varepsilon_1'(c'_i), \varepsilon_2'(c'_i))$ (denoted by 5 dots for each figure) and the corresponding fitness $F'_i$ in various generations. Figure 7 shows the variations in the maximum and average fitness values defined by Eq. (16) with generations. Figure 6 shows that the individuals' $(\varepsilon_1, \varepsilon_2)$ that are initially distributed at random in the range of $(\varepsilon_1, \varepsilon_2) \in \{L_{\varepsilon1} \leq \varepsilon_1 \leq U_{\varepsilon1}, L_{\varepsilon2} \leq \varepsilon_2 \leq U_{\varepsilon2}\}$ gather in the domain of $\{(\varepsilon_1, \varepsilon_2) | 200 \leq \varepsilon_1 \leq 300, 400 \leq \varepsilon_2 \leq 500\}$. In addition, $F'_{\text{max}}$, the largest length of the downward lines in each figure, increases with the number of generations. By inspecting Figs 6 and 7, the solution to the problem is found to be $(\varepsilon_1, \varepsilon_2) \equiv (271.7, 437.0)$ which gives $f(\varepsilon_1, 0.6, 0.05; \varepsilon_2, 0.7, 0.05) \equiv 7.316$. The solution is obtained from $2^{10} \times 2^{10} (=1048576)$ candidates of $(\varepsilon_1, \varepsilon_2)$, as shown in Eqs (13) and (18); however, the calculation (the number of individuals (5) multiplied by the number of generations (50)) to obtain Fig. 7 is performed only 900 times. Thus, the computational cost is reduced to $1/4194 (\equiv 250/(2^{10} \times 2^{10}))$ of the cost of 100% inspection, which also shows an even greater advantage for a solution search using the genetic algorithm.
3.6. Optimization with Combination of Tensioning Intensities and Locations

In the previous subsections, the optimization problems with respect to the combinations of 2 parameters, namely \((\ell_1,\ell_2)\) and \((\ell_1',\ell_2')\), were solved. Because the natural frequency of the most critical mode is dependent on 6 parameters \((\ell_1, t_1, b_1, \ell_2, t_2, b_2)\) as described by Eq. (11), such a higher dimensional optimization problem is solved in this subsection.

Once the rollers used for tensioning are introduced in a tensioning site, it is hard to modify the shape of rollers, i.e., tensioning widths \(b_1\) and \(b_2\). In practice, the variable parameters are the intensities and locations \((\ell_1, t_1, \ell_2, t_2)\). Therefore, the optimization problem described by Eq. (11) is solved with respect to the combination of \((\ell_1, t_1, \ell_2, t_2)\), by setting \(L_{b1} = U_{b1} = 0.05\) and \(L_{b2} = U_{b2} = 0.05\). The selected parameters are...
\[
\begin{align*}
L_{e1} &= 0, U_{e1} = 1000, n_{e1} = 10; \quad L_r = 0.325, U_r = 0.975, n_r = 10; \\
L_{e2} &= 0, U_{e2} = 1000, n_{e2} = 10; \quad L_r = 0.325, U_r = 0.975, n_r = 10; \\
N_{\text{population}} &= 50, \ N_{\text{elit}} = 2; \quad p_{\text{crossover}} = 0.7, \ p_{\text{mutation}} = 0.05
\end{align*}
\] (19)

Figure 8 shows the variations in the maximum and average fitness values defined by Eq. (16) from the 0th through to 160th generation, in which sufficient convergence of the maximum fitness value on \(F_{\text{max}}^{160} \approx 7.426\) is achieved. The combination of parameters to give the converged maximum fitness value is found to be \((\lambda_1, \lambda_2, \lambda_3, \lambda_4) \approx (514.2, 0.7393, 416.4, 0.5531)\). The solution is obtained from \(2^{10} \times 2^{10} \times 2^{10} \times 2^{10} \approx 1.100 \times 10^{12}\) candidates of \((\lambda_1, \lambda_2, \lambda_3, \lambda_4)\), as shown in Eqs (13) and (19); however, the calculation (the number of individuals (50) multiplied by the number of generations (160)) to obtain Fig. 8 is performed only 8000 times. Thus, the computational cost is reduced to \(1/(137.4 \times 10^6)\) \((\approx 8000/(2^{10} \times 2^{10} \times 2^{10} \times 2^{10}))\) of the cost of 100% inspection, which strongly confirms the advantage of searching for a solution using the genetic algorithm.

![Figure 8: Variations in maximum and average fitness values with generation \((b_1 = b_2 = 0.05)\)](image)

4. Conclusions:

In this study, we used a genetic algorithm to solve the optimization problem for the tensioning parameters in a rotating circular saw under a thermal load that is tensioned over a double annular domain. We first presented an analytical model in which rotation, local temperature due to friction, and in-plane plastic strain owing to tensioning were considered. Next, we obtained the analytical solution for the in-plane forces and carried out modal analysis for the flexural vibration. We then applied the genetic algorithm to the optimization problem for stable operation of the saw to maximize the natural frequency of the most critical mode. Initially, the algorithm was applied only to an optimization problem where one of the tensioning locations was varied, and it was found that the genetic algorithm not only functioned properly but also required much lower computing costs than 100% inspection. Next, the genetic algorithm was applied to an optimization problem with two tensioning locations or two tensioning intensities. Lastly, the algorithm was applied to an even higher dimensional optimization problem, that is, a problem with two tensioning locations and two tensioning intensities. Optimal tensioning parameters were obtained in both the two-variable and four-variable cases; this resulted in computational costs that were considerably lower than those required for 100% inspection.
References